DIFFERENTIAL GEOMETRY I FINAL EXAMINATION

Total marks: 50
Attempt all questions
Time: 3 hours (10 am - 1 pm)

(1) Consider the parametrized surface (for $(u,v) \in \mathbb{R}^2$)
$$\phi(u,v) = (u - \frac{u^3}{3} + uv^2, v - \frac{v^3}{3} + vu^2, u^2 - v^2).$$
Compute the coefficients of the first and second fundamental forms. Compute the principal curvatures. Show that the lines of curvature are the coordinate curves. Show that the asymptotic curves are $u + v = const$ and $u - v = const$. (6+4+4+4 = 18 marks)

(2) True or False: (a) All meridians of a torus are geodesics. (b) All meridians of a torus are lines of curvature. Justify both answers. (5+5 = 10 marks)

(3) State and justify whether the sphere and the surface $z = x^2 - y^2$ are locally isometric or not. (8 marks)

(4) True or false: Let S be the cylinder $x^2 + y^2 = 1$. Given any two points p and q on S, and any two unit tangent vectors $v \in T_pS$ and $w \in T_qS$, there exists a piecewise regular parametrized curve C in S joining p to q such that the parallel translate of v at p along C is w at the point q. Justify your answer. (5 marks)

(5) State the local and global Gauss-Bonnet Theorems. Prove that any two simple closed geodesics (that is, closed regular curves which are geodesics) on a compact connected oriented regular surface of positive curvature must intersect. (6+3 = 9)

Date: 13th November, 2019.